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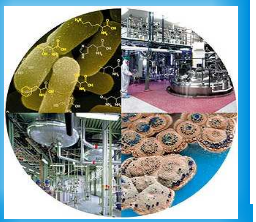

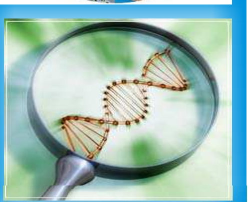

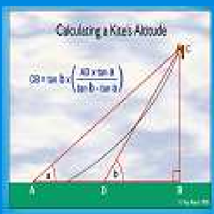

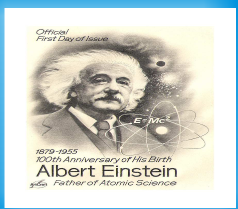

BHU - MCA

SAMPLE THEORY

- **NUMERICAL INTEGRATION**
- **NUMBER SERIES**

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A. NUMERICAL INTEGRATION

Number Integration

The process of evaluation of definite integral $\int_a^b f(x)\,dx$ for the set of numerical values of the integrand f (x) is known as numerical integration. When this is used for function of a single variable, then this is called ''quadrature''.

Important Formula

1. Trapezoidal rule

$$\int_{x_0}^{x_0+nh} ydx = h\left[\frac{1}{2}(y_0+y_n)+(y_1+y_2+...,+y_{n-1})\right]$$

2. Error in Trapezoidal Rule

Starting with the interval x, to x_o + h and proceeding parallel to the case (A), we find that the inherent error in the trapezoidal rule is given by

$$E_r = -\frac{h^3}{12}[f''(x_o)+f''(x_1)+.......+f''(x_{n-1})]$$

where $x_o = a$, $x_o + nh = b$ and $x_i = x_o + ih$.

If $f''(\xi) = \max |f''(x)|$, $a<\xi<b$. $a\le x\le b$

we may write (ii) as

$$E_r \le -\frac{h^3 n}{12} f''(\xi) = \frac{(b-a)^3}{12n^2} f''(\xi) = -\frac{(b-a)}{12} h^2 f''(\xi)$$

3. Simpson's one third $\left(\frac{1}{3}\right)$ rule

$$\int_{x_0}^{x_0+nh} ydx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

4. Error in Simpson's One-third Rule

Consider a function f (x) which is finite and continuous in the interval $(x_. - h, x_. + h)$ and has continuous derivatives of all orders upto and including the fourth in that interval. Further, let

$$F(x)= \int_a^x f(x)dx$$

so that F'(x) = f(x), F"(x) = f' (x), etc.

Now

$$I = \int_{x_0-h}^{x_0+h} f(x)dx = F(x_0+h) - F(x_0-h)$$

By Simpson's one-third rule

$$I_s = \frac{h}{3}[f(x_0-h) + 4\ f\ (x_0) + f(x_0+h)]$$

hence the inherent error in the Simpson's one-third rule is given by

$$E_s = -\frac{h^5}{90} f^{iv}(x_0), = -\frac{(b-a)}{180} h^4\ f^{iv}(x_0)$$

Solved Example

1 **Evaluate the following integral by Simpson's 1/3 method**

Sol. Dividing the range of integration (0, 6) into 6 equal parts, $\frac{6-0}{6} = 1 = h$

The values of the function f (x) = $\frac{1}{1+x^2}$ at each point of division are given in the following table.

	x_0	x_0+h	x_0+2h	x_0+3h	x_0+4h	x_0+5h	x_0+6h
x_i:	0	1	2	3	4	5	6
y_i:	1.00	.5	.2	0.100	0.058824	.038462	.027027

Table

By Simpson's 1/3 Rule

$$I = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+.027027)+4(.5+0.100+.038462)+2(.2+.058824)\right] = 1.366174$$

2 From the following table find the value of x for which f(x) is a maximum. Also find the maximum value of f(x) from the table of values given below. Using string 's formula

x	60	75	90	105	120
f(x)	28.2	38.2	43.2	40.9	37.7

(A) x = 92.1135, maximum value is 43.27

(B) x = 90.1135, maximum value is 43.27

(C) x = 90.1135, maximum value is 40.27

(D) x = 92.1135, maximum value is 40.27

2.(A) The maximum value appears to be in the neighbourhood of x = 90. Hence we will use Stirling's formula.

[redacted]

By Stirling's formula

$$y(x) = y_0 + \frac{t}{2}\left(\Delta y_0 + \Delta y_{-1}\right) + \frac{t^2}{2}\Delta^2 y_{-1} +$$

$$\frac{t\left(t^2 - 1^2\right)}{12}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right) + \frac{t^2\left(t^2 - 1^2\right)}{24}\Delta^4 y_{-2} + ...$$

$$y = 43.2 + \frac{t}{2}(-2.3 + 5) + \frac{t^2}{2}(-7.3) + \left(\frac{t^3 - t}{12}\right)(-2.3 + 6.4)$$

$$= 0.341u^3 - 3.65u^2 + 1.0083u + 43.2$$

If y is maximum $\frac{dy}{dx} = 0$

i.e. $3(0.3417)t^2 - 2(3.65)t + 1.0083 = 0$

$1.025lt^2 - 7.3t + 1.0083 = 0$

$$t = \frac{7.3 \pm \sqrt{(7.3)^2 - 4(1.0251)(1.0083)}}{2 \times 1.0251} = t = \frac{7.3 \pm 7.0111}{2.0502} = 6.9803(\text{or})0.1409$$

t = 6.9803 goes beyond the range

$\therefore$ t = 0.1409

$x = x_0 + th = 90 + 15\,(0.1409) = 92.1135$

$\text{Maximum} = 0.3417(0.1409)^3 - 3.65(0.1409)^2$

$+ 1.0083(0.1409) + 43.2 = 43.27$

$\therefore$ (x) is maximum at x = 92.1135 and the maximum value is 43.27

B. NUMBER SERIES

This part consist of question in which terms of a number series are given. These terms follow a certain pattern throughout the series. Questions based on this can broadly be divided into three types.

Type 1 : In this type of series a number pattern is given. The candidate is required to study the given number series, identify the pattern and find the missing term.

Solved Example

Direction: Find the missing number in the following :

1. 5, 25, 125, 625, ?

(A) 3145

(B) 3125

(C) 3165

(D) 3115

1.(B) The pattern is 5^1, 5^2, 5^3 and 5^4 . So, ? = 5^5 = 3125.

Type 2 : In this type of series a number pattern is given. The candidate is required to study the given number series, identify the pattern and identify the term in the number series which is wrong.

Solved Example

Direction: Identify the wrong number in the following number series :

1. 5, 7, 11, 19, 36, 67, 131

(A) 7

(B) 36

(C) 11

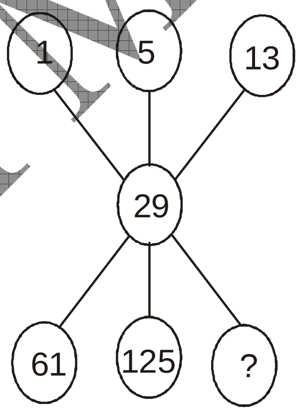
(D) 67

1.(B) The pattern is 5 + 2 = 7, 7 + 4 = 11, 11 + 8 = 19, 19 + 16 = 35, 35 + 32 = 67 and 67 + 64 = 131. So, 36 is wrong.

Type 3: In such type of questions, a figure, a set of figures is given which follow a certain pattern. The candidate is required to find the correct pattern and fill in the missing numbers accordingly.

Solved Example

1. Which number replaces the question mark?



(A) 257

(B) 255

(C) 251

(D) 253

1.(D) Starting at the top left, and moving through the diagram in a Z shape, double each number and add 3 to get the next number along.

$\Rightarrow$ $? = 125 \times 2 + 3 = 253$